VISCOSITY IN A HETEROGENEOUS LIQUID-SOLIDS SYSTEM

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As a result of an experimental and theoretical investigation a formula has been derived for the viscosity in a heterogeneous liquid-solids system. This formula is valid over the entire range of concentrations of the solid phase.

Solid particles in a flow create local turbulent perturbations that increase the viscosity of the fluid. A similar effect is observed in free turbulent flows [1] as a result of velocity fluctuations. The presence of velocity fluctuations in heterogeneous fluid-solids system and their parameters were experimentally established in [2].

Investigations have shown that in a bed of solid particles suspended by an ascending flow of water the mean values of the velocities and velocity fluctuations do not vary over the cross section of the flow and that the relative magnitude of the velocity fluctuations $\Delta\bar{v}/\bar{v}$ is proportional to the volume concentration of solids. The homogeneity and localization of turbulence are attributable to the fact that the particles form a porous medium dividing the flow into local components and suppressing the natural turbulence of the flow as a whole.

A similar effect is created by grids in wind tunnels and in the motion of water saturated with solids. In the first case the grids reduce the degree of turbulence by as much as 1000 times [1]. In the second case the flow regime remains more or less laminar at Reynolds number up to 400 000 [3].

In a suspended bed with a fixed upper boundary the particles are in a certain state of random motion. This produces the effect known as dynamic concentration. The motion of the particles in the bed does not have any other effect on the flow structure or parameters. (This was established by parallel experiments with and without circulation of the solid particles.) Therefore in determining the laws of motion of water in a suspended bed it is usual to assume dynamic equilibrium and a uniform distribution of particles over the volume of the bed [4, 5].

The ordinary viscosity of a fluid is associated with molecular-kinetic effects, the laws of which were used by Prandtl in deriving a formula for the turbulent viscosity of free flows. The nature of these effects consists in a transfer of momentum between the fluid particles in the shear plane.

The increase of viscosity in turbulent flows and in a heterogeneous system is due to the transfer of momentum, as a result of turbulent perturbations, between "macroscopic, more or less large elements (moles) of the fluid" [1]. In this case, near solid surfaces the molecular viscosity intensifies the turbulent perturbations [6]. Consequently, the viscosity in a heterogeneous system may be assumed proportional to the molecular viscosity μ_0 and to the increase in the momentum of unit mass of the fluid as it moves in the variable sections between solid particles* (as compared with the motion of a fluid with constant mean velocity v [7]).

Hence

$$\mu_m = \mu_0\left(\frac{p+\Delta p}{p}\right) = \mu_0\left(1+\frac{\Delta p}{p}\right). \qquad (1)$$

Since

$$p = mv, \text{ and } \frac{\Delta p}{p} = \alpha_k \frac{\bar{v}_{\max} - \bar{v}_{\min}}{\bar{v}},$$

we have

$$\mu_m = \mu_0\left(1+\alpha_k \frac{\bar{v}_{\max} - \bar{v}_{\min}}{\bar{v}}\right) = \mu_0\left(1+\alpha_k \frac{\Delta\bar{v}}{\bar{v}}\right). \qquad (2)$$

The quantities in Eq. (2) are given by

$$\bar{v} = \frac{v_0}{\omega}, \quad \bar{v}_{\max} = \frac{v_0}{\bar{\omega}_{\min}}, \quad \bar{v}_{\min} = \frac{v_0}{\bar{\omega}_{\max}},$$

where

$$\omega = 1 - C_0,$$

$$\bar{\omega}_{\min} = 1 - n^{2/3}\frac{\pi d^2}{4} \approx (1-C_0)^2,$$

$$\bar{\omega}_{\max} = 2\omega - \bar{\omega}_{\min} = 1 - C_0^2.$$

Whence

$$\frac{\bar{v}_{\max} - \bar{v}_{\min}}{\bar{v}} = \frac{\Delta\bar{v}}{\bar{v}} = 2C_0. \qquad (3)$$

The coefficient α_k is usually [8] determined from the expressions

$$\alpha_k = \frac{\int_\omega (\bar{v}+\Delta v)^2\, d\omega}{\bar{v}^2\omega} = \left(\frac{\bar{v}_{\max}}{\bar{v}}\right)^2. \qquad (4)$$

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*Experimental determinations of the viscosity of suspensions confirm this proportionally, which is reflected in all the formulas for this viscosity. A suspended layer of solid particles is also a suspension.

Replacing $\bar{v}_{max}$ and $\bar{v}$ in (4) by the expressions presented above gives

$$\alpha_k = \frac{1}{(1 - C_0)^2}. \tag{5}$$

After substituting the values of $\Delta\bar{v}/\bar{v}$ and α_k from (3) and (4) into Eq. (2), we obtain

$$\mu_m = \mu_0 \left[1 + \frac{2C_0}{(1 - C_0)^2} \right]. \tag{6}$$

The values of μ_m given by Eq. (6) are too low as compared with the experimental data. The reason for this is that in a heterogeneous system as compared with the assumed concentration of the stationary particles the constriction of the section and the nonuniformity of the velocities are increased: in the suspended bed owing to circulation of the particles (dynamic concentration) and in the fixed bed owing to the stagnant zones in the part of the pore space between grains.

With allowance for this additional constriction of the flow cross section C_d the mean minimum area $\bar{\omega}'_{min} = (1 - C_0 - C_d)^2$, and the mean maximum velocity

$$\bar{v}_{max} = \frac{v_0}{(1 - C_0 - C_d)^2}. \tag{7}$$

In this case

$$\alpha_k = \left(\frac{\bar{v}'_{max}}{\bar{v}} \right)^2 = \left[\frac{1 - C_0}{(1 - C_0 - C_d)^2} \right]^2. \tag{8}$$

An analysis of the values of C_d obtained in experiments on a suspended bed [2]* and analytically for a fixed bed [18] has shown that

$$\left(\frac{\bar{v}'_{max}}{\bar{v}} \right)^2 \approx \left(\frac{\bar{v}_{max}}{\bar{v}_{min}} \right)^2. \tag{9}$$

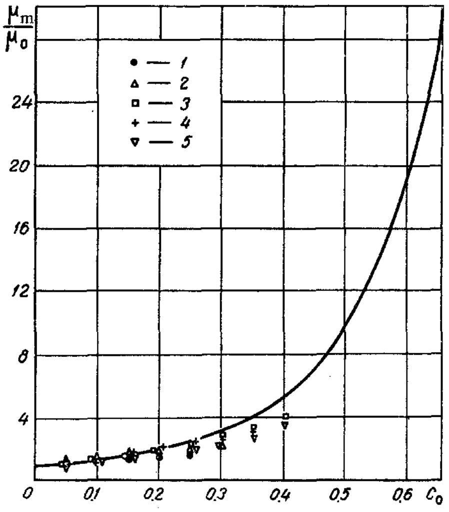
Using the simpler expression on the right side of (9), we write

$$\alpha_k = \left(\frac{\bar{v}_{max}}{\bar{v}_{min}} \right)^2 = \left(\frac{1 + C_0}{1 - C_0} \right)^2. \tag{10}$$

Then

$$\mu_m = \mu_0 \left[1 + 2C_0 \left(\frac{1 + C_0}{1 - C_0} \right)^2 \right]. \tag{11}$$

The phenomenon of molar viscosity as a consequence of momentum transfer between macromasses of fluid applies equally to the suspended and the fixed bed, since in both cases transfer takes place as a result of continuous sharp changes in the velocities of the elementary flow components. This is confirmed by the successful extrapolation of the theory and laws of motion of a fluid in a fixed bed to a suspended bed. The latter is regarded as an intermediate state between the fixed filter bed and the free fall of an individual particle [4, 9, 10]. Moreover, it has been established [11] that the use of the molar viscosity in formulas for the filtration of water in a granular bed makes it possible to eliminate the empirical coefficients, obtain laws corresponding to the experimental data, and theoretically determine the boundaries of the filtration regimes.



The ratio μ_m/μ_0 as a function of the particle concentration C_0 according to formula (11) and the formulas of: 1) Einstein; 2) Hacek; 3) Wand; 4) Manley and Malone; 5) Eulers.

A comparison of experiments with moving and stationary particles in a suspended bed has shown that these particles do not participate directly in the momentum transfer. Therefore the molar viscosity, both dynamic and kinematic, relates directly to the liquid phase and does not necessitate taking into account the apparent density of the heterogeneous system as a whole. The experiments of Broughton [12] and Trawiński [13] with mixtures containing particles with density equal to or greater than that of the liquid phase lead to a similar conclusion. This also follows from the assumed physical model of the effect. Therefore

$$\frac{\mu_m}{\mu_0} = \frac{\nu_m}{\nu_0}. \tag{12}$$

The results of calculations based on Eq. (11) are presented in Table 1, which will serve as a calculating aid, like ordinary molecular viscosity tables. The figure is a graphic expression of relation (11), showing the nature of the variation of the molar viscosity as the volume concentration increases. The same figure includes points whose ordinates were obtained from the formulas of other authors [14]. These formulas give satisfactory agreement with the experimental data on narrow intervals of values of C_0 and are a special case of formula (11).

Table 2 presents experimental data on the viscosity in a liquid-solids heterogeneous system obtained by Broughton [12] and Trawinski [13].

Remark. Trawinski performed experiments with an aqueous suspension of lime, Broughton with pellets

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*The maximum value C_d = 0.11 corresponds to C_0 = = 0.25–0.3.

Table 1

Molar Viscosity as a Function of Particle Concentration

C_0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7
μ_m/μ_0	1.125	1.3	1.55	1.9	2.39	3.07	4.02	5.36	7.25	10.0	14.05	20.2	29.9	46.1
	Suspended bed											Fixed bed		

Table 2

Results of Experimental Determinations of Molar Viscosity

μ_m/μ_0	C_0					
	0.05	0.1	0.15	0.2	0.25	0.3
Acc. to Broughton	1.15	1.35	1.62	1.94	—	—
Acc. to Trawinski	1.17	1.35	1.67	2.10	2.6	4.10

having the same density as the liquid dispersion medium.

A comparison of Tables 1 and 2 shows that the experiments confirm the theoretical values of the viscosity calculated from (11). A certain discrepancy observed at $C_0 > 0.25$ is evidently attributable to imperfections in the method of experimental determination of the viscosity of a liquid-solids mixture. In particular, in A. M. Mostkov's experiments [15], owing to these imperfections the viscosity tended to infinity at $C_0 = 0.35$, which contradicts reality. At the same time in Zhukovitskii's experiments [16] with a clay suspension at $C_0 \approx 0.6$ a value $\mu_m/\mu_0 = 1 + 30\, C_0 = 19$ was obtained. This is very close to the value presented in Table 1.

Formula (11) was also checked against experimental data on the settling of particle systems at volume concentrations from 0.07 to 0.6 [4, 5, 17].

NOTATION

$\bar{v}$ is the mean velocity of the water in a heterogeneous system; $\Delta\bar{v}$ is the mean value of the velocity fluctuations; μ_0 is the molecular viscosity of the water; μ_m is the molar viscosity of the water; p is the momentum of the water flow elements with mean velocity $\bar{v}$; Δp is the momentum increment due to velocity fluctuations; m is the mass of the water flow element; $\bar{v}_{max}$ is the mean maximum velocity of the water in a heterogeneous system; $\bar{v}_{min}$ is the mean minimum velocity of the water in a heterogeneous system; v_0 is the velocity of the water referred to a unit cross-sectional area of a heterogeneous system; α_k is the flow momentum coefficient characterizing the nonuniformity of the velocity distribution; ω is the mean useful flow section in a particle medium; $\bar{\omega}_{min}$ is the mean minimum area of useful section of the local flow component in a particle medium; $\bar{\omega}_{max}$ is the mean maximum area of useful section of the local flow component in a particle medium; Δv is the velocity fluctuation; C_0 is the volume concentration of particles in a liquid medium; n is the number of particles per unit volume of a heterogeneous system; d is the particle diameter; ν_0 is the kinematic viscosity of the liquid phase in a heterogeneous system; ν_m is the molar kinematic viscosity of the liquid phase in a heterogeneous system; C_d is the additional constriction of the flow section as a result of particle circulation in the suspended bed or stagnant zones of a fixed bed; $\bar{\omega}'_{min}$ is the mean minimum area between particles, with allowance for C_d; $\bar{v}'_{max}$ is the mean maximum velocity of water between particles, with allowance for C_d.

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